Discrete Mathematics

Quiz # 1 (March XX, 2015)

Name:______________________________ID:__________________________________

1. (30%) Which of the following is(are) logically equivalent to $\neg(p \vee q) \leftrightarrow (q \rightarrow r)$?
   (There might be more than one correct answer.)
   (a) $(q \wedge r) \vee \neg(p \vee q)$
   (b) $(\neg(p \vee q) \rightarrow (q \rightarrow r)) \wedge ((q \rightarrow r) \rightarrow \neg(p \vee q))$
   (c) $(q \wedge \neg r) \wedge \neg(p \vee q)$
   (d) $(q \wedge \neg r) \vee \neg(p \vee q)$
   (e) $(\neg(p \vee q) \rightarrow (p \rightarrow r)) \wedge ((q \rightarrow r) \rightarrow \neg(p \vee q))$

Ans: b d e

2. (10%) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. (True or False)
   (a) $\forall x \exists y((2x + 3y = 2) \wedge (x + 1.5y = 1))$
   Ans: True

   (b) $\neg\forall x \exists y(x^2 \geq y)$
   Ans: False

3. (30%) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

Ans:

1. $\exists x \neg P(x)$
2. $\neg P(c)$ from (1)
3. $\forall x(P(x) \vee Q(x))$
4. $P(c) \vee Q(c)$ from (3)
5. $Q(c)$ from (2) and (4)
6. $\forall x(\neg Q(x) \vee S(x))$
7. $\neg Q(c) \vee S(c)$ from (6)
8. $S(c)$ from (5) and (7)
9. $\forall x(R(x) \rightarrow \neg S(x))$
10. $R(c) \rightarrow \neg S(c)$ from (9)

11. $\neg R(c)$ from (8) and (10)

12. $\exists x \neg R(x)$ from (11)

4. (30%) Use a proof by contradiction to show that there is no rational number *r* for which $r^3 + r + 1 = 0$.

Ans:

Assume that there exist a rational number r=a/b, where gcd(a,b)=1.

Then we can obtain the equation $a^3 + ab^2 + b^3 = 0$.

If a and b are both odd, the left-hand side is the sum of three odd numbers and therefor must be odd.

If a is even and b is odd (or a is odd and b is even), then the left-hand side is odd + even + even, which is again odd.

Hence a and b are both even but it is contradicted to the original assumption: gcd(a,b)=1. This contradiction shows that no such root exists.